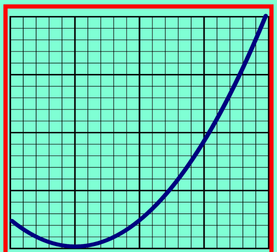
Math 125

Fall 2021

Lecture 28



$y = ax^2 + bx + c$

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Consider

$$\begin{cases} 5x + 3y = 7 \\ -x + 2y = 9 \end{cases}$$

Use Cramer's rule to find y only.

$$\boxed{y = \frac{D_y}{D}}$$

$$D = \begin{vmatrix} 5 & 3 \\ -1 & 2 \end{vmatrix} = 5(2) - (-1)(3) = 10 + 3 = \boxed{13}$$

$$D_y = \begin{vmatrix} 5 & 7 \\ -1 & 9 \end{vmatrix} = 5(9) - (-1)(7) = 45 + 7 = \boxed{52}$$

$$y = \frac{D_y}{D} = \frac{52}{13} = \boxed{4}$$

$$\boxed{y = 4}$$

Given

$$\begin{cases} x - 2y + z = 4 \\ 3x + y - 2z = 3 \\ 5x + 5y + 3z = -8 \end{cases}$$

Use Cramer's Rule to Solve for z only.

$$z = \frac{D_z}{D}$$

$$D = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 5 & 5 & 3 \end{vmatrix} = 1\begin{vmatrix} 1 & -2 \\ 5 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 3 & -2 \\ 5 & 3 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 5 & 5 \end{vmatrix}$$

$$= 1(3-(-10)) + 2(9-(-10)) + 1(15-5)$$

$$= 1 \cdot 13 + 2 \cdot 19 + 1 \cdot 10 = \boxed{61}$$

$$D_z = \begin{vmatrix} 1 & -2 & 4 \\ 3 & 1 & 3 \\ 5 & 5 & -8 \end{vmatrix} = 1\begin{vmatrix} 1 & 3 \\ 5 & -8 \end{vmatrix} - (-2)\begin{vmatrix} 3 & 3 \\ 5 & -8 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ 5 & 5 \end{vmatrix}$$

$$= 1(-8-15) + 2(-24-15) + 4(15-5)$$

$$= -23 - 78 + 40$$

$$= \boxed{-61}$$

$$z = \frac{D_z}{D} = \frac{-61}{61} = -1 \qquad \boxed{z = -1}$$

Solving System of linear equations by Matrix Method:

1) Set-up Augmented matrix
2) Use elementary row operations to change all element above and below the main digonal Zero. We like to make main diagonal elements to be all 1's.

$$\left[\begin{array}{ccc|c} 1 & & 0 & \\ & 1 \; \cdots & & \\ 0 & & 1 & \end{array}\right]$$

③ Final Ans is all the numbers on the last Column.

Elementary Row operations:

1) Interchange any two rows.
2) Multiply/Divide any row by a non-Zero number.
3) Add any multiple of any row to any other row to replace that row.

$$\begin{cases} x - y = 5 \\ 2x + 3y = -10 \end{cases}$$

$\Rightarrow$ Augmented Matrix is a matrix made of Coef.'s and RHS numbers Separated by dotted line.

$$\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 2 & 3 & -10 \end{array}\right]$$

$(-2)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 5 & -20 \end{array}\right]$$

$(R2) \div 5 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 0 & 1 & -4 \end{array}\right]$$

$R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -4 \end{array}\right]$$

$x = 1, \ y = -4$

$\{(1,-4)\}$ $\boxed{(x,y) = (1,-4)}$

$$\begin{cases} -3x + 2y = 21 \\ x - y = -8 \end{cases}$$

$\Rightarrow$ Augmented Matrix

$$\left[\begin{array}{cc|c} -3 & 2 & 21 \\ 1 & -1 & -8 \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & -8 \\ -3 & 2 & 21 \end{array}\right]$$

$(3)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & -8 \\ 0 & -1 & -3 \end{array}\right]$$

$(-1)R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -1 & -8 \\ 0 & 1 & 3 \end{array}\right]$$

$R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right]$$

$x = -5$

$y = 3$ $\Rightarrow (-5, 3)$

Solve by matrix method:

$$\begin{cases} y = 2x-2 \\ x = 11-2y \end{cases} \Rightarrow \begin{cases} -2x + y = -2 \\ x + 2y = 11 \end{cases}$$

Hint: make Sure everything is lined up.

Augmented Matrix

$$\left[\begin{array}{cc|c} -2 & 1 & -2 \\ 1 & 2 & 11 \end{array}\right] \xrightarrow{\text{use elementary Row operations}} \left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \end{array}\right]$$

$R1 \leftrightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & 11 \\ -2 & 1 & -2 \end{array}\right]$$

$(2)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & 11 \\ 0 & 5 & 20 \end{array}\right]$$

$R2 \div 5 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & 2 & 11 \\ 0 & 1 & 4 \end{array}\right]$$

$(-2)R2 + R1 \rightarrow R1$

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \end{array}\right] \quad \begin{array}{l} x = 3 \\ y = 4 \end{array} \Rightarrow \boxed{(3,4)}$$

Solve by matrix Method:

$$\begin{cases} x - 2y - 3 = 0 \\ 2x - 4y - 7 = 0 \end{cases} \qquad \begin{cases} x - 2y = 3 \\ 2x - 4y = 7 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 2 & -4 & 7 \end{array}\right]$$

$(-2)R1 + R2 \rightarrow R2$

$$\left[\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 0 & 1 \end{array}\right]$$

when one row is made of all Zeros

$\Rightarrow$ infinite number of Solutions

when one row is made of all Zeros except the number on the last column

$\Rightarrow$ $\boxed{\text{There are no Solutions}}$

No Solution

The difference of two Complementary angles is 10°. Use matrix method to find both angles.

$$\begin{cases} x + y = 90 \\ x - y = 10 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 90 \\ 1 & -1 & 10 \end{array}\right]$$

$(-1)R1 + R2 \to R2$

$$\left[\begin{array}{cc|c} 1 & 1 & 90 \\ 0 & -2 & -80 \end{array}\right]$$

$(R2) \div (-2) \to R2$

$$\left[\begin{array}{cc|c} 1 & 1 & 90 \\ 0 & 1 & 40 \end{array}\right]$$

$(-)R2 + R1 \to R1$

$$\left[\begin{array}{cc|c} 1 & 0 & 50 \\ 0 & 1 & 40 \end{array}\right]$$

$x = 50$

$y = 40$

$\Rightarrow \boxed{50^\circ \,\&\, 40^\circ}$

Solve by matrix Method

$$\begin{cases} x + y + z = 5 \\ 2x - y + z = -3 \\ 3x + 2z = 2 \end{cases} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & -1 & 1 & -3 \\ 3 & 0 & 2 & 2 \end{array}\right]$$

$(-2)R1 + R2 \to R2$

$(-3)R1 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -3 & -1 & -13 \\ 0 & -3 & -1 & -13 \end{array}\right]$$

$(-1)R2 + R3 \to R3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -3 & -1 & -13 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

One Row made of All Zeros

$\boxed{\text{infinite \# of Solutions}}$

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

System of 3 linear equations in 3 variables.

$$\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \boxed{} \\ 0 & 1 & 0 & \boxed{} \\ 0 & 0 & 1 & \boxed{} \end{array}\right]$$

x y z

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Given

$$\begin{cases} 3x - 2y + z = 16 \\ 2x + 3y - z = -9 \\ x + 4y + 3z = 2 \end{cases}$$

Find D, the determinant of coef. matrix.

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 4 & 3 \end{vmatrix} = 3\begin{vmatrix} 3 & -1 \\ 4 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= 3(9+4) + 2(6+1) + 1(8-3)$$

$$= 3\cdot 13 + 2\cdot 7 + 1\cdot 5 = \boxed{58}$$